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# Haar Wavelet based Numerical Method for the Solution of Non-Linear Boundary Value Problems Arising in Fluid Dynamics

**S. C. Shiralashetti***

**Abstract**

In this paper, haar wavelet based numerical method is developed for the solution of non-linear boundary value problems arising in fluid dynamics. An operational matrix of integration based on the Haar wavelet is established, and the procedure for applying the matrix to solve these equations, which satisfies the boundary conditions, is formulated. The fundamental idea of Haar wavelet method is to convert the proposed differential equations into a group of non-linear algebraic equations, which involves a finite number of variables are solved using Inexact Newton's method. The nonlinear boundary value problem (BVP) for the electrohydrodynamic flow of a fluid in an ion drag configuration in a circular cylindrical conduit is considered and obtained the numerical solutions based on the haar wavelet based numerical method for various values of the relevant parameters ( $\alpha$, $Ha$ ) and also obtained the residual and square residual errors. Comparisons are made between the MTLAB (bvp4c) method of solution and the proposed method of solution. It is shown that the MTLAB (bvp4c) method of solution is equivalent to the haar wavelet based numerical method of solution. The accuracy of approximate solution can be further improved by increasing the level of resolution and an error analysis is computed. The example is given to demonstrate the fast and flexibility of the method. The results obtained are in good agreement with the existing ones and it is shown that the technique introduced here is robust, easy to apply and is not only enough accurate but also quite stable.

***Keywords:***

Haar Wavelet Collocation Method, Electrohydrodynamic Flow, Non-Linear Boundary Value Problem.

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## 1. Introduction

The electrohydrodynamic flow of a fluid in an ion drag configuration in a circular cylindrical conduit was first reviewed by McKee [15]. In that article, a full description of the problem was presented in which the governing equations were reduced to the nonlinear boundary value problem (BVP)

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}+Ha^2\left(1-\frac{y}{1-\alpha y}\right)=0,\quad 0<x<1 \tag{1}$$

subject to the boundary conditions

$$y'(0)=0,\quad y(1)=0 \tag{2}$$

where $y(x)$ is the fluid velocity, $x$ is the radial distance from the center of the cylindrical conduit, $Ha$ is the Hartmann electric number, and the parameter $\alpha$ is a measure of the strength of the nonlinearity. Perturbative and numerical solutions to (1) and (2) for small/large values of $\alpha$ were provided. Paullet [16] proved the existence and uniqueness of a solution to (1) and (2), and in addition, discovered an error in the perturbative and numerical solutions given in [15] for large values of $\alpha$ .

Beginning in the 1980s, wavelets have been used for solution of ordinary and partial differential equations. The good features of this approach are the possibility to detect singularities, irregular structure and transient phenomena exhibited by the analyzed equations. Most of the wavelet algorithms can handle exactly periodic boundary conditions. The wavelet algorithms for solving ordinary and partial differential equations are based on the Galerkin techniques or on the collocation method. Evidently, all attempts to simplify the wavelet solutions for ODE and PDE are welcome. One possibility for this is to make use of the Haar wavelet family. Haar wavelets (which are Daubechies of order one) consist of piecewise constant functions and are therefore the simplest orthonormal wavelets with a compact support. A drawback of the Haar wavelets is their discontinuity. Since the derivatives do not exist in the breaking points, it is not possible to apply the Haar wavelets for solving PDE directly. There are two possibilities for getting out of this situation. One way is to regularize the Haar wavelets with interpolating splines (e.g. B-splines or Deslaurier-Dabuc interpolating wavelets). This approach has been applied by Cattani [4]. The other way is to make use of the integral method, which was proposed by Chen and Hsiao [5]. Recently, Lepik [11-13] established the Haar wavelet method for solving some ODEs and PDEs. There are some useful discussions by other researchers [1-3 & 7-9]. The purpose of this present work is to present accurate numerical solutions to (1) and (2) for all values of the relevant parameters using the haar wavelets Islam et al. [10]. Haar wavelet based numerical method (HWNM) for solving nonlinear boundary value problems arising in electrohydrodynamic flow, which will exhibit several advantageous features: (i) Very high accuracy fast transformation and possibility of implementation of fast algorithms compared with other known methods. (ii) The simplicity and small computation costs, resulting from the sparsity of the transform matrices and the small number of significant wavelet coefficients. (iii) The method is also very convenient for solving the boundary value problems, since the boundary conditions are taken care of automatically.

The paper is organized in the following way. For completeness, the properties of Haar wavelets are given in Section 2. Haar Wavelet based numerical method of solution is presented in Section 3. Numerical experiments are presented in Section 4. Conclusions are given in Section 5.

## 2. Properties of Haar Wavelets

The scaling function $h_1(x)$ for the family of the Haar wavelets is defined as

$$h_1(x) = \begin{cases} 1 & for \quad x \in [1,\, 0) \\ 0 & otherwise \end{cases} \tag{3}$$

The Haar wavelet family for $x \in [1,\, 0)$ is defined as

$$h_i(x) = \begin{cases} 1 & for \quad x \in \left[\dfrac{k}{m},\, \dfrac{k+0.5}{m}\right) \\ -1 & for \quad x \in \left[\dfrac{k+0.5}{m},\, \dfrac{k+1}{m}\right) \\ 0 & otherwise \end{cases} \tag{4}$$

In the above definition the integer, $m = 2^l,\ l = 0, 1, \ldots, J$, indicates the level of resolution of the wavelet and integer $k = 0, 1, \ldots, m-1$ is the translation parameter. Maximum level of resolution is $J$. The index $i$ in Eq. (4) is calculated using, $i = m + k + 1$. In case of minimal values $m = 1,\ k = 0$, then $i = 2$. The maximal value of $i$ is $K = 2^{J+1}$. Let us define the collocation points $x_j = \dfrac{j-0.5}{K},\ \ j = 1, 2, \ldots, K$, discretize the Haar function $h_i(x)$ and the corresponding Haar coefficient matrix $H(i,\, j) = (h_i(x_j))$, which has the dimension $K \times K$.

The following notations are introduced

$$PH_{1,\,i}(x) = \int_0^x h_i(x)\,dx \tag{5}$$

$$PH_{2,\,i}(x) = \int_0^x PH_{1,\,i}(x)\,dx \tag{6}$$

These integrals can be evaluated by using Eq. (4), first and 2nd operational matrices are as follows,

$$PH_{1,i}(x)=\begin{cases} x-\dfrac{k}{m} & for \quad x\in\left[\dfrac{k}{m},\dfrac{k+0.5}{m}\right) \\ \dfrac{k+1}{m}-x & for \quad x\in\left[\dfrac{k+0.5}{m},\dfrac{k+1}{m}\right) \\ 0 & otherwise \end{cases} \tag{7}$$

and

$$PH_{2,i}(x)=\begin{cases} \dfrac{1}{2}\left(x-\dfrac{k}{m}\right)^2 & for \quad x\in\left[\dfrac{k}{m},\dfrac{k+0.5}{m}\right) \\ \dfrac{1}{4m^2}-\dfrac{1}{2}\left(\dfrac{k+1}{m}-x\right)^2 & for \quad x\in\left[\dfrac{k+0.5}{m},\dfrac{k+1}{m}\right) \\ \dfrac{1}{4m^2} & for \quad x\in\left[\dfrac{k+1}{m},1\right) \\ 0 & otherwise \end{cases} \tag{8}$$

We also introduce the following notation

$$CH_{1,i}=\int_0^1 PH_{1,i}(x)dx \tag{9}$$

Any function $f(x)$ which is square integrable in the interval (0, 1) can be expressed as an infinite sum of Haar wavelets as

$$f(x)=\sum_{i=1}^{\infty}a_i h_i(x) \tag{10}$$

The above series terminates at finite terms if $f(x)$ is piecewise constant or can be approximated as piecewise constant during each subinterval.

The best way to understand wavelets is through a multi-resolution analysis. Given a function $f\in L_2(\mathbb{R})$ a multi-resolution analysis (MRA) of $L_2(\mathbb{R})$ produces a sequence of subspaces $V_j, V_{j+1},\ldots$ such that the projections of *f* onto these spaces give finer and finer approximations of the function *f* as $j\to\infty$.

**Multi-resolution Analysis:** A multi-resolution analysis of $L_2(\mathbb{R})$ is defined as a sequence of closed subspaces $V_j\subset L_2(\mathbb{R}),\ j\in\mathbb{Z}$ with the following properties

(i) $\ldots\subset V_{-1}\subset V_0\subset V_1\subset\ldots$

(ii) The spaces $V_j$ satisfy $\bigcup_{j\in\mathbb{Z}}V_j$ is dense in $L_2(\mathbb{R})$ and $\bigcap_{j\in\mathbb{Z}}V_j=0$.

(iii) If $f(x)\in V_0,\ f(2^j x)\in V_j$, i.e. the spaces $V_j$ are scaled versions of the central space $V_0$.

(iv) If $f(x)\in V_0,\ f(2^j x-k)\in V_j$ i.e. all the $V_j$ are invariant under translation.

(v) There exists $\phi\in V_0$ such that $\phi(x-k);\ k\in\mathbb{Z}$ is a Riesz basis in $V_0$.

The space $V_j$ is used to approximate general functions by defining appropriate projection of these functions onto these spaces. Since the union of all the $V_j$ is dense in $L_2(\mathbb{R})$, so it guarantees that any function in $L_2(\mathbb{R})$ can be approximated arbitrarily close by such projections. As an example the space $V_j$ can be defined like

$$V_j=W_j\oplus V_{j-1}=W_{j-1}\oplus W_{j-2}\oplus V_{j-2}=\ldots=\bigoplus_{j=1}^{J+1}W_j\oplus V_0$$

then the scaling function $h_j(x)$ generates an MRA for the sequence of spaces $\{V_j,\ j\in\mathbb{Z}\}$ by translation and dilation as defined in Eqs. (3) and (4). For each $j$ the space $W_j$ serves as the orthogonal complement of $V_j$ in $V_{j+1}$. The space $W_j$ include all the functions in $V_{j+1}$ that are orthogonal to all those in $V_j$ under some chosen inner product. The set of functions which form basis for the space $W_j$ are called wavelets [6, 14].

## 3. Haar Wavelet based Numerical Method of Solution

In this section, we apply HWNM to solve (1) and (2) for the fluid velocity $y(x)$, we assume that

$$y''(x)=\sum_{i=1}^{K}a_i h_i(x) \tag{11}$$

Eq. (11) is integrated twice from 0 to $x$ or from $x$ to 1 depending upon the boundary conditions. Hence the solution $y(x)$ with its derivatives $y'(x)$ and $y''(x)$ are expressed in terms of the Haar functions and their integrals. The expressions $y(x)$, $y'(x)$ and $y''(x)$ are substituted in the given differential equation and discretization is applied using the collocation points $x_j$ resulting into a $K\times K$ linear or nonlinear system. The haar coefficients $a_i,\ i=1,2,\ldots,K$ are calculated by solving this system. The approximate solution can easily be recovered with the help of the Haar coefficients in the form of MRA.

Integrate Eq. (11) and using boundary conditions we can express $y'(x)$ and $y(x)$ as

$$y'(x)=\beta+\sum_{i=1}^{K}a_i PH_{1,i}(x) \tag{12}$$

$$y(x)=\tilde{y}(x)=\gamma-\beta(1-x)-\sum_{i=1}^{K}a_i(CH_{1,i}-PH_{2,i}) \tag{13}$$

where $\beta=y'(0),\quad \gamma=y(1)$

Substitute $y''(x),\ y'(x)$ and $y(x)$ (Eqs. (11) - (13)) in Eq. (1), we get

$$\sum_{i=1}^{K}a_i h_i(x)+\frac{1}{x}\left(\beta+\sum_{i=1}^{K}a_i PH_{1,i}(x)\right)+Ha^2\left[1-\frac{\gamma-\beta(1-x)-\sum_{i=1}^{K}a_i(CH_{1,i}-PH_{2,i})}{1-\alpha\left(\gamma-\beta(1-x)-\sum_{i=1}^{K}a_i(CH_{1,i}-PH_{2,i})\right)}\right]=0 \tag{14}$$

Solve Eqn. (14) using Newton inexact method, we get haar wavelet collocation coefficients $a_i$'s, in Table 1. Then substitute these coefficients in Eq. (13), we get the required HWCM numerical solution $y(x)$.

To facilitate the analysis in the next section, we substitute (13) into (1) to obtain the residual function.

$$R(x)=\frac{d^2\tilde{y}}{dx^2}+\frac{1}{x}\frac{d\tilde{y}}{dx}+Ha\left(1-\frac{\tilde{y}}{1-\alpha\tilde{y}}\right) \tag{15}$$

We also define the square residual error for the N[th] order approximation to be,

$$E_N=\int_0^1\left[R(x)\right]^2dx. \tag{16}$$

## 4. Numerical Experiments

Here we solve (1) with (2) numerically using the MATLAB (bvp4c) and compare with the HWCM based numerical solutions obtained in the previous section and is presented in Table 2 for $N=16, \alpha=0.5,1,\ Ha^2=4,10$, using,

$$h(16,16)=\begin{pmatrix}
1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1\\
1&1&1&1&1&1&1&1&-1&-1&-1&-1&-1&-1&-1&-1\\
1&1&1&1&-1&-1&-1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&1&-1&-1&-1&-1\\
1&1&-1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&-1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&-1&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&-1&-1\\
1&-1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&-1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&-1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&-1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&-1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&-1
\end{pmatrix},$$

$$PH_{1,i}(x)=PH_1(16,16)=\frac{1}{32}\begin{pmatrix}
1&3&5&7&9&11&13&15&17&19&21&23&25&27&29&31\\
1&3&5&7&9&11&13&15&15&13&11&9&7&5&3&1\\
1&3&5&7&7&5&3&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&3&5&7&7&5&3&1\\
1&3&3&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&3&3&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&3&3&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&3&3&1\\
1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1
\end{pmatrix}$$

$$PH_2(16,16)=\frac{1}{2048}\begin{pmatrix}
1 & 9 & 25 & 49 & 81 & 121 & 169 & 225 & 289 & 361 & 441 & 529 & 625 & 729 & 841 & 961\\
1 & 9 & 25 & 49 & 81 & 121 & 169 & 225 & 287 & 343 & 391 & 431 & 463 & 487 & 503 & 511\\
1 & 9 & 25 & 49 & 79 & 103 & 119 & 127 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 25 & 49 & 79 & 103 & 119 & 127\\
1 & 9 & 23 & 31 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 1 & 9 & 23 & 31 & 32 & 32 & 32 & 32 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 23 & 31 & 32 & 32 & 32 & 32\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 9 & 23 & 31\\
1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7 & 8 & 8\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 7
\end{pmatrix}$$

&

$$CH_1(16,16)=\frac{1}{256}\begin{pmatrix}
128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 128\\
128 & 128 & 128 & 128 & 128 & 128 & 128 & 128 & 64 & 64 & 64 & 64 & 64 & 64 & 64 & 64\\
128 & 128 & 128 & 128 & -16 & -16 & -16 & -16 & 16 & 16 & 16 & 16 & 16 & 16 & 16 & 16\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & 32 & 32 & 16 & 16 & 16 & 16\\
128 & 128 & -68 & -68 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4\\
0 & 0 & 0 & 0 & 72 & 72 & -28 & -28 & 4 & 4 & 4 & 4 & 4 & 4 & 4 & 4\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & 32 & -4 & -4 & 4 & 4 & 4 & 4\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & 8 & 4 & 4\\
128 & -97 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & 98 & -71 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 72 & -49 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 50 & -31 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 32 & -17 & 1 & 1 & 1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 18 & -7 & 1 & 1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 8 & -1 & 1 & 1\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1
\end{pmatrix}$$

**Table 1. Haar wavelet collocation coefficients $a_i$'s of Eq. (14).**

| $\alpha = 0.5,\ Ha = 4$ | $\alpha = 1,\ Ha = 10$ |
|---|---|
| -1.3615 | -4.383 |
| 0.40654 | -6.8993 |
| 0.06278 | 0.14619 |
| 0.37705 | -4.0836 |
| 0.0121 | 0.02913 |
| 0.05464 | 0.12858 |
| 0.13763 | -10.49 |
| 0.23548 | 8.18647 |
| 0.00267 | 0.00684 |
| 0.00977 | 0.02282 |
| 0.02014 | 0.04582 |
| 0.03518 | 0.086 |
| 0.05606 | 0.17348 |

| | |
|---|---|
| 0.08201 | -21.681 |
| 0.108 | 5.3828 |
| 0.12606 | 2.45886 |

And also numerical findings are presented in Fig. 1 for $N = 128, \alpha = 1, Ha^2 = 10.$, subsequently figs. 2 to 6 present the numerical findings for $N = 128$ and different values of $\alpha = 0.05, 0.5, 1, 5$ and $Ha^2 = 1, 2, 4, 10, 15, 25$, which shows the efficiency of the HWCM.

The residual and square residual errors are presented in Table 3 for different values of $N$, $\alpha$ and $Ha$, subsequently Fig. 7 to 10 presents the residual errors for $N = 128,\ Ha^2 = 0.5, 1, 2, 4\ \ \alpha = 0.5, 1$, which shows the convergence of the HWNM numerical solution. These numerical experiments demonstrate that the HWNM numerical solutions for various values of the relevant parameters compare extremely well with the MATLAB (bvp4c) numerical solutions. For all of the cases considered, the maximum difference between the HWNM and the MATLAB (bvp4c) numerical solutions was determined to be less than $10^{-3}$.

Table 2: Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 16$.

| | $\alpha = 0.5,\ Ha = 4$ | | $\alpha = 1,\ Ha = 10$ | |
|---|---|---|---|---|
| x | HWNM | MATLAB (bvp4c) | HWNM | MATLAB (bvp4c) |
| 0.03125 | 0.48222 | 0.48285 | 0.48976 | 0.49039 |
| 0.09375 | 0.48015 | 0.48078 | 0.48921 | 0.48984 |
| 0.15625 | 0.47477 | 0.47539 | 0.48772 | 0.48836 |
| 0.21875 | 0.46626 | 0.46687 | 0.48521 | 0.48584 |
| 0.28125 | 0.45451 | 0.4551 | 0.48138 | 0.48201 |
| 0.34375 | 0.43931 | 0.43988 | 0.4758 | 0.47642 |
| 0.40625 | 0.42037 | 0.42092 | 0.46784 | 0.46845 |
| 0.46875 | 0.39735 | 0.39786 | 0.45658 | 0.45718 |
| 0.53125 | 0.3698 | 0.37028 | 0.4408 | 0.44137 |
| 0.59375 | 0.33724 | 0.33767 | 0.41884 | 0.41938 |
| 0.65625 | 0.29909 | 0.29948 | 0.38854 | 0.38905 |
| 0.71875 | 0.25472 | 0.25505 | 0.34721 | 0.34766 |
| 0.78125 | 0.20343 | 0.2037 | 0.29157 | 0.29195 |
| 0.84375 | 0.14444 | 0.14463 | 0.21788 | 0.21816 |
| 0.90625 | 0.07693 | 0.07703 | 0.12208 | 0.12224 |
| 0.96875 | 0 | 0 | 0 | 0 |

Table 3: The residual and square residual errors.

| $\alpha = 0.5$ | | | | | | |
|---|---|---|---|---|---|---|
| | $H^2 = 1$ | | $H^2 = 2$ | | $H^2 = 4$ | |
| N | Minimum value of Residual | Minimum value of $E_N$ | Minimum value of Residual | Minimum value of $E_N$ | Minimum value of Residual | Minimum value of $E_N$ |
| 4 | 6.189E-4 | 3.831E-7 | 1.493E-4 | 2.229E-8 | 9.541E-5 | 9.102E-9 |
| 16 | 4.189E-5 | 2.347E-11 | 2.421E-7 | 5.86E-14 | 6.733E-6 | 3.478E-15 |
| 32 | 9.203E-7 | 6.04E-13 | 3.115E-8 | 9.703E-16 | 2.639E-8 | 1.734E-17 |
| 64 | 1.087E-9 | 2.25E-16 | 5.633E-11 | 1.567E-20 | 6.878E-10 | 5.451E-21 |
| 128 | 5.367E-10 | 9.477E-17 | 5.031E-12 | 1.966E-23 | 3.417E-13 | 1.4E-24 |
| $\alpha = 1$ | | | | | | |
| 4 | 2.978E-4 | 8.867E-8 | 8.761E-5 | 7.675E-9 | 5.445E-6 | 3.592E-11 |
| 16 | 7.548E-7 | 5.697E-13 | 3.276E-8 | 1.073E-15 | 1.473E-7 | 1.783E-12 |
| 32 | 2.198E-8 | 9.167E-15 | 4.826E-10 | 2.329E-19 | 6.014E-9 | 3.616E-15 |
| 64 | 1.477E-10 | 2.182E-18 | 1.104E-11 | 2.854E-21 | 2.785E-12 | 7.758E-19 |
| 128 | 4.113E-11 | 1.691E-20 | 1.416E-12 | 6.647E-23 | 2.587E-14 | 3.376E-23 |

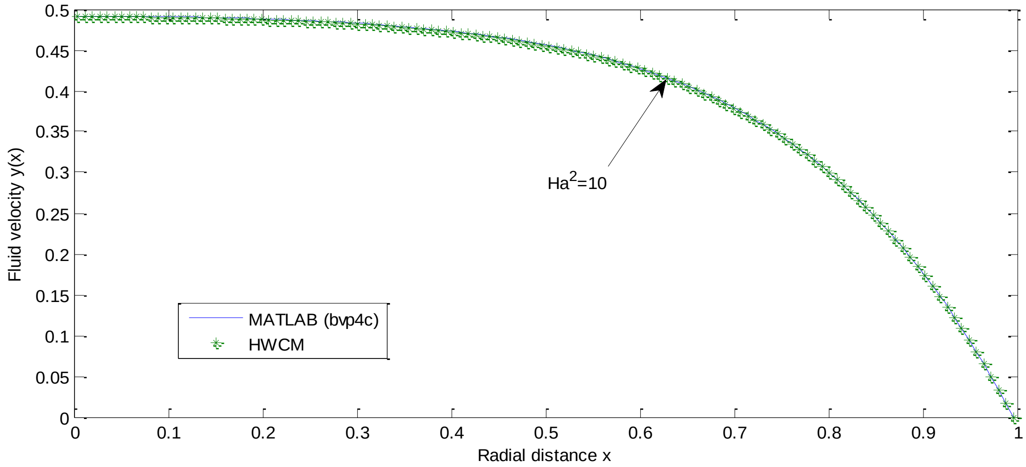



Fig. 1. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 1$.

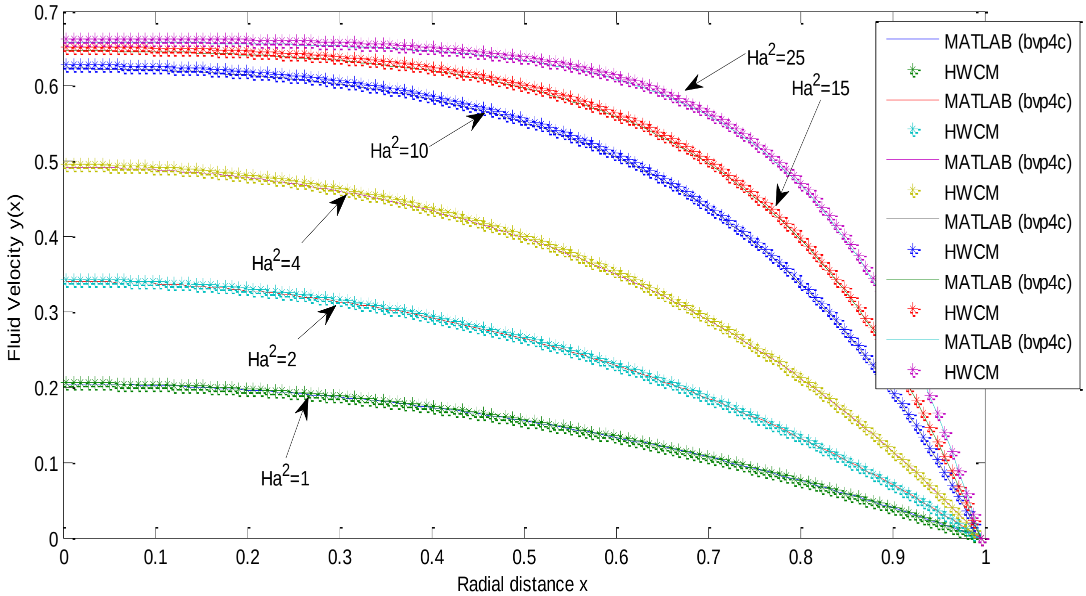



Fig. 2. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 0.05$.

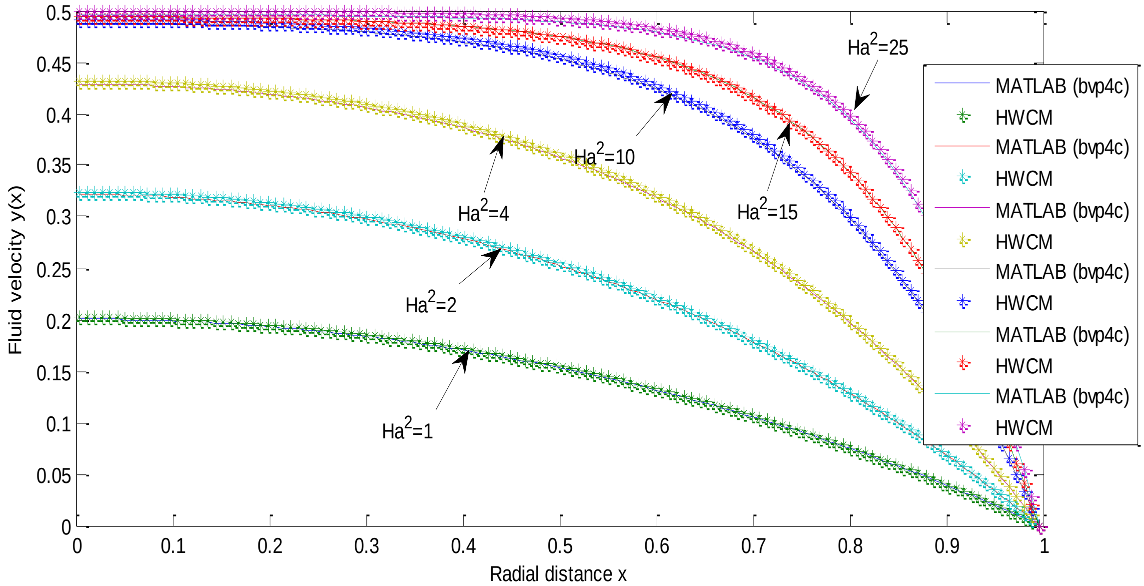



Fig. 3. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 0.5$.

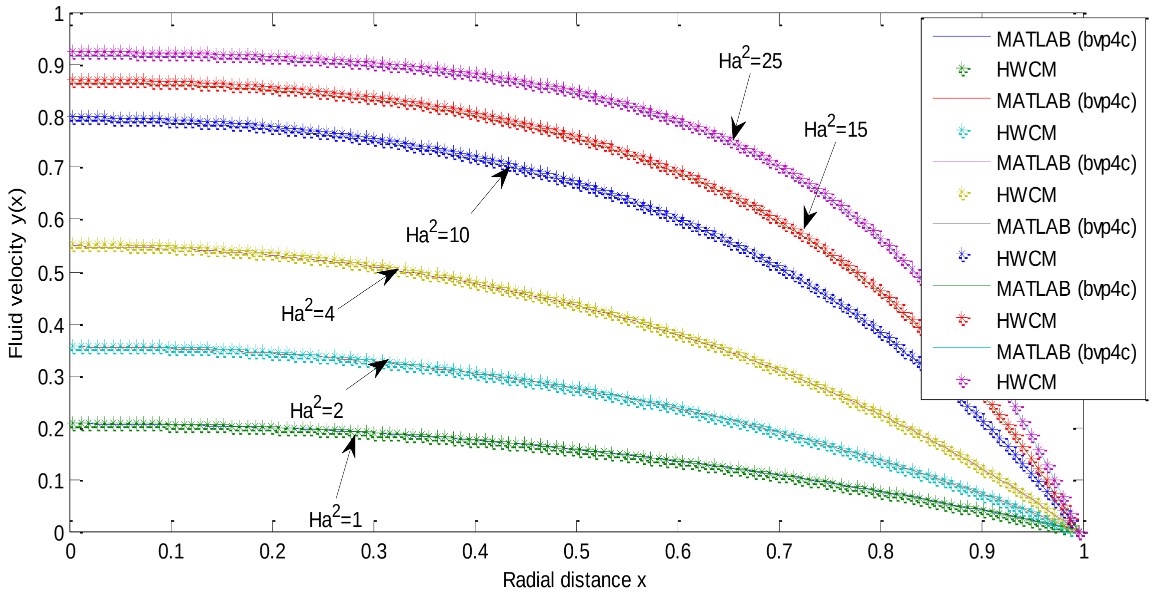



Fig. 4. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 1$.

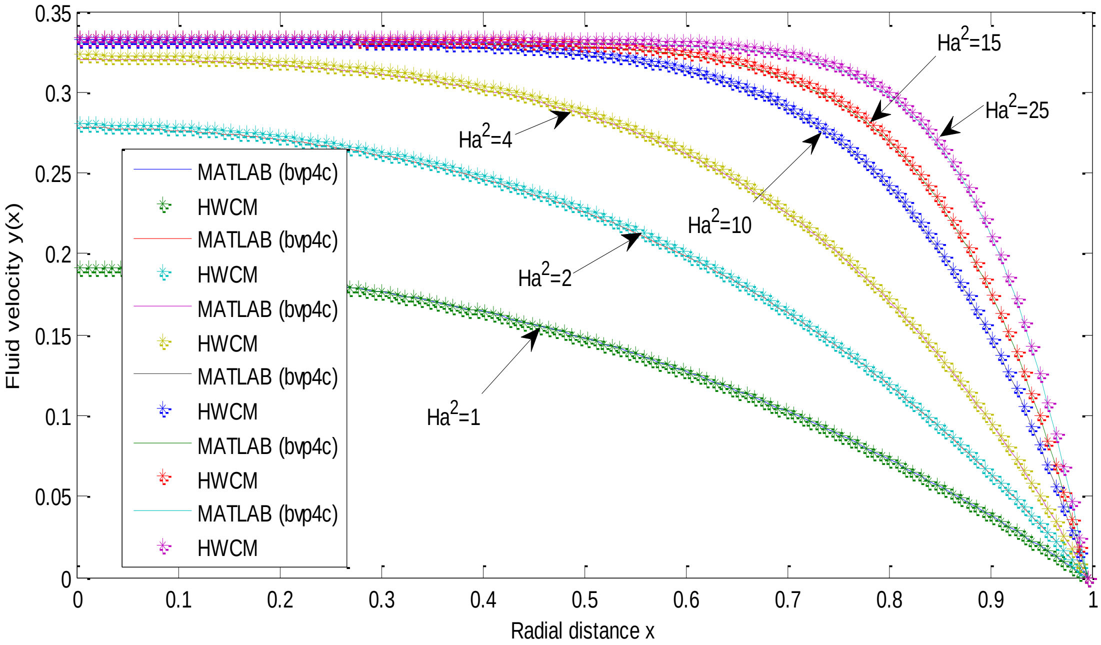



Fig. 5. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 2$.

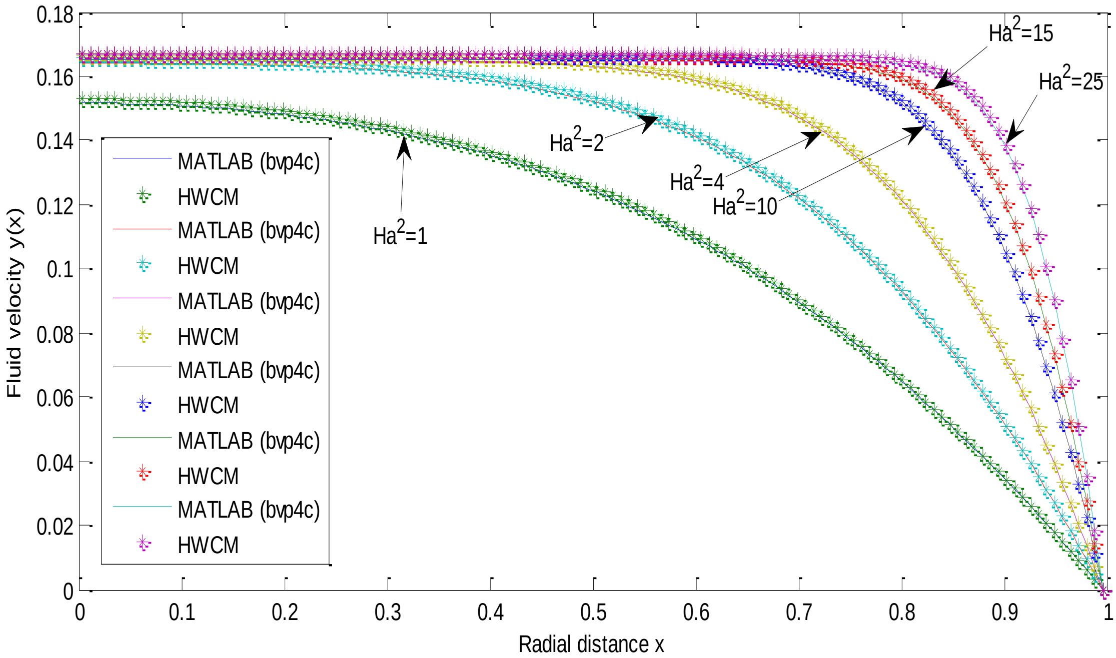
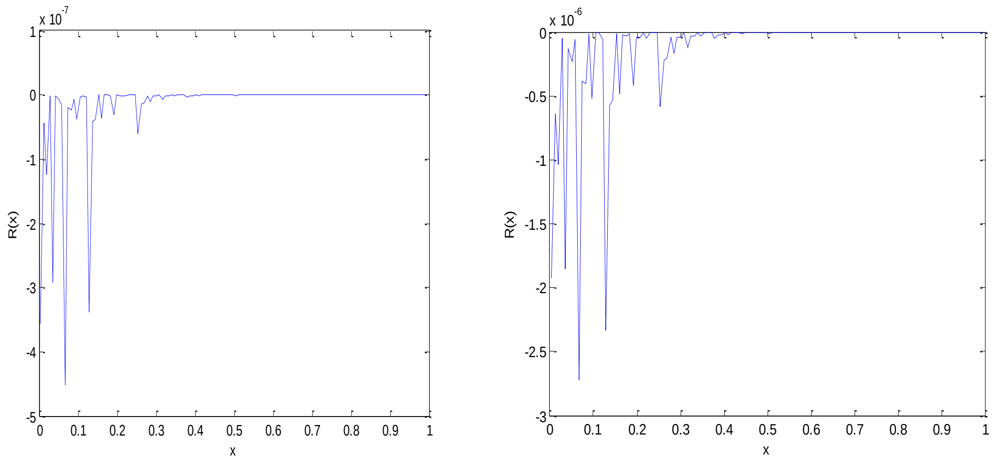
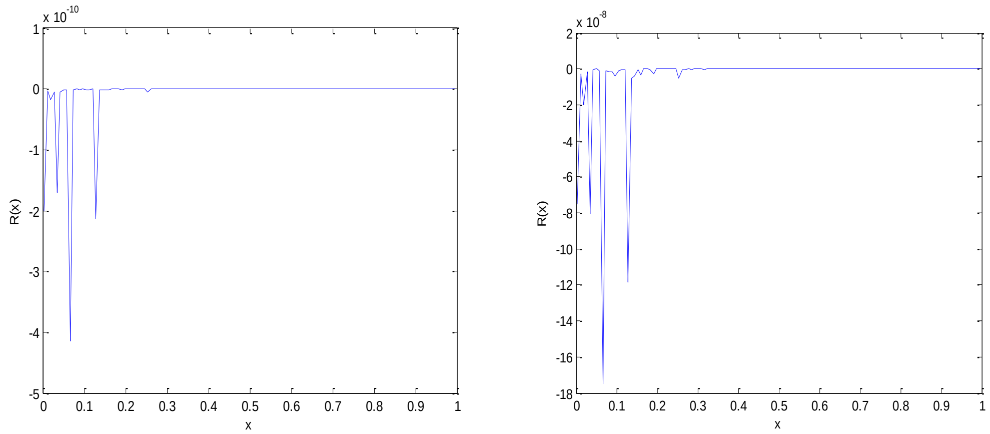



Fig. 6. Comparison of numerical solution MATLAB (bvp4c) with HWNM for $N = 128,\ \alpha = 5$.



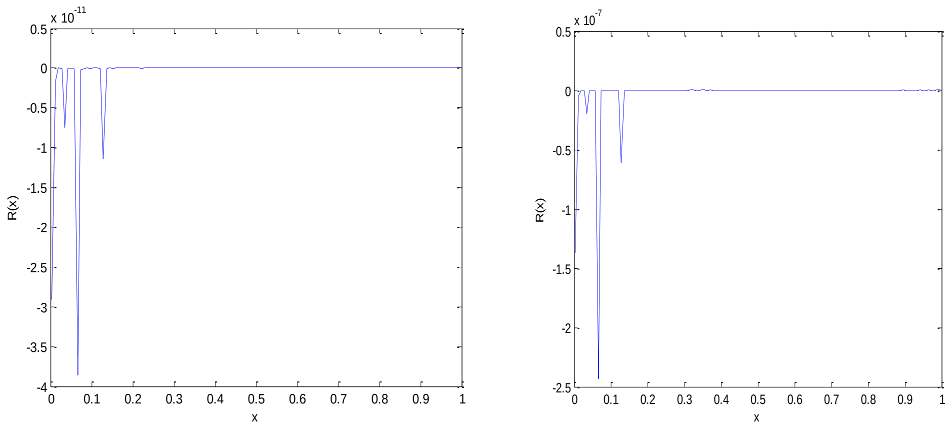

(a) (b)

Fig. 7. The residual of HWNM for $N = 128$, $Ha^2 = 0.5$ and (a) $\alpha = 0.5$, (b) $\alpha = 1$.



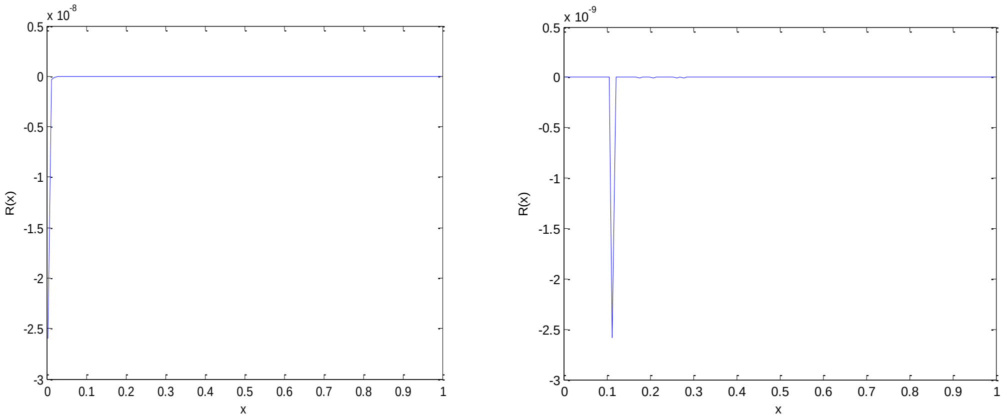

(a) (b)

Fig. 8. The residual of HWNM for $N = 128$, $Ha^2 = 1$ and (a) $\alpha = 0.5$, (b) $\alpha = 1$.




(a) (b)

Fig. 9. The residual of HWNM for $N = 128$, $Ha^2 = 2$ and (a) $\alpha = 0.5$, (b) $\alpha = 1$.




(a) (b)

Fig. 10. The residual of HWNM for $N=128$, $Ha^2=4$ and (a) $\alpha=0.5$, (b) $\alpha=1$.

## 5. Conclusions

In this paper, the haar wavelet based numerical method (HWNM) has been developed for the numerical solutions of nonlinear boundary value problems arising in fluid dynamics in particularly electrohydrodynamic flow. It has been noted that the nonlinearity confronted in this problem is in the form of a rational function, and thus, poses a significant challenge in regard to obtaining numerical solutions. Despite this fact, shown that the HWNM based numerical solutions obtained are convergent and that they compare extremely well with MATLAB (bvp4c) based numerical solutions. It is also shown that the HWNM yields divergent solutions for all of the cases considered. These results demonstrate that HWNM is a very effective numerical method for solving highly nonlinear problems arising in science and engineering.

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